Eg ① (1993)

(a) A particle of mass m moves with simple harmonic motion under the action of a variable force. If the maximum value of the force is $\frac{7m}{16}$ and the amplitude of the motion is 4 m calculate

(i) the period of the oscillation.

(ii) the speed of the particle at a time $\frac{2\pi}{\sqrt{7}}$ seconds after passing through the centre of oscillation.

(b) A light elastic string, of elastic constant $\frac{48mg}{l}$ and natural length l has one end attached to a fixed point. Two particles of masses $3m$ and $2m$ are attached to the other end and the system hangs in equilibrium. If the $2m$ mass falls off

(i) prove that the $3m$ mass will move with simple harmonic motion of period

$$\frac{\pi}{2}\sqrt{\frac{l}{g}}$$

(ii) find the amplitude of the motion.

ⓐ NII ⇒ Nett $f = ma$

SHM ⇒ Force $= m\omega^2 x$ about $x = 0$

Told ⇒ $\frac{7}{16}m = m\omega^2 A$ [Max force at extreme]

$\Rightarrow \frac{7}{16}m = m\omega^2 4$

$\Rightarrow \frac{7}{64} = \omega^2 \Rightarrow \omega = \frac{\sqrt{7}}{8}$ Rad/sec

(i) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\sqrt{7}}{8}} = \frac{16\pi}{\sqrt{7}}$ seconds.

(ii) Find v

$x = 0$ at O, $x = ?$ at P, $t = \frac{2\pi}{\sqrt{7}}$ sp. $\boxed{v = ?}$

First x:

$x = A\sin\omega t$

$x = 4\sin\left(\frac{\sqrt{7}}{8}\frac{2\pi}{\sqrt{7}}\right)$

$x = 4\sin\left(\frac{\pi}{4}\right)$

$x = 4\frac{1}{\sqrt{2}}$

$x = 2\sqrt{2}$.

Now v.

$v^2 = \omega^2(A^2 - x^2)$

$v^2 = \left(\frac{\sqrt{7}}{8}\right)^2\left(4^2 - (2\sqrt{2})^2\right)$

$v^2 = \frac{7}{64}(16 - 8)$

$v^2 = \frac{7}{64}(8)$

$v^2 = \frac{7}{8} \Rightarrow v = \sqrt{\frac{7}{8}} = .94$ secs

OR (ii) $x = A\sin\omega t$

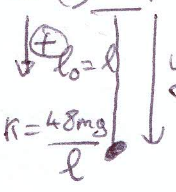
$\Rightarrow v = \frac{dx}{dt} = +A\omega\cos\omega t$

$\Rightarrow v = 4\left(\frac{\sqrt{7}}{8}\right)\cos\left[\left(\frac{\sqrt{7}}{8}\right)\left(\frac{2\pi}{\sqrt{7}}\right)\right]$

$v = \frac{\sqrt{7}}{2}\cos\left(\frac{\pi}{4}\right)$

$v = \frac{\sqrt{7}}{2}\cdot\frac{1}{\sqrt{2}} = \sqrt{\frac{7}{8}}$ as before

ⓑ (i) Let equil be y below ceiling. $\boxed{a = 0}$



Forces: S, $3mg$

Hooke $|S| = \frac{48mg}{l}(y - l)$

NII $(a=0)$ ⇒ $-|S| + 3mg = 0$

$\Rightarrow -\frac{48mg}{l}(y - l) + 3mg = 0$

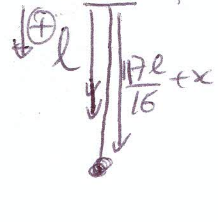
$\Rightarrow -\frac{48y}{l} + 48 + 3 = 0$

$\Rightarrow 48y = 51l$

$\Rightarrow \boxed{y = \frac{17l}{16}}$

Equil is $\frac{17l}{16}$ below ceiling.

Examine forces at $\frac{17l}{16} + x$ below ceiling.



Forces: S', $3mg$

Hooke $S' = k(\text{Ext})$

$S' = \frac{48mg}{l}\left(\frac{l}{16} + x\right)$

NII: $\Sigma F = ma$

$\Rightarrow -\frac{48mg}{l}\left(\frac{l}{16} + x\right) + 3mg = 3ma$

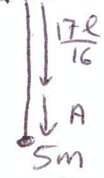
$\Rightarrow -3mg - \frac{48mg}{l}x + 3mg = 3ma$

$\Rightarrow \underline{-\frac{16g}{l}x = a}$

SHM with $\boxed{\omega = 4\sqrt{\frac{g}{l}}}$

$\therefore T = \frac{2\pi}{\omega} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$ qed.

(ii) Find A. Find where particles started from ie where 2m mass drops off and where accel $= \omega^2 A = \left(4\sqrt{\frac{g}{l}}\right)^2 A$



Forces: T'', $5mg$

Hooke $T'' = k\left(\frac{17l}{16} + A - l\right)$

$T'' = \frac{48mg}{l}\left[\frac{l}{16} + A\right]$

NII ⇒ $\Sigma F = Ma$

$\Rightarrow -\frac{48mg}{l}\left[\frac{l}{16} + A\right] = 5m\left[\left(4\sqrt{\frac{g}{l}}\right)^2\right]A$

$\Rightarrow A = \frac{l}{24}$.